Micromechanics of Quartz Sand Breakage in a Fractal Context

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Abstract:
From a Quaternary science perspective, sand-sized quartz as well as silt-sized quartz is often acknowledged as final products of glacial abrasion through different evolution mechanisms. This view challenges the existence of any universal comminution process, which may relate the formation of detrital quartz sand and silt. The contribution of grain size, energy input, and crystalline integrity in the scale of quartz crushability has long been matter of much debate. The present empirical work examines the micromechanics of sand-to-silt size reduction in the quartz material. A series of grinding experiments was performed on Leighton Buzzard Lower Greensand using a high-energy disc mill. Analogous conditions to glacial abrasion are provided due to the combined abrasion between grains’ asperity tips, and also between grains and rotating smooth tungsten carbide pestle. Discontinuous breakage approach allowed a control on grains’ crystalline defects. To enable an objective assessment of micromechanics of size reduction, measurements of particle and mode size distribution, fractal indexes and micro-morphological signatures were made. The crushing approach was probed through varied grinding time at a constant energy input, as well as varied energy input at constant grinding time. Breakage pathway was inspected via laser diffraction spectroscopy and transmission light microscopy. Results suggested that the grain breakdown is not necessarily an energy-dependent process. Non-crystallographically pure quartz sand and silt are inherently breakable materials through a fractal breakdown process. Results also revealed that the internal defects in quartz are independent from size and energy input.

Key words:
Quartz; breakage; fractal; micromechanics; glacial

1. Introduction
Loess, for which silt is the main constituent, is formed through cycles of Quaternary geologic (e.g. crystallization of magma), geomorphic (e.g. solifluction or cryoturbation) and climatic (e.g. thermal or chemical weathering) processes (Smalley et al., 2006 b). A good understanding of loess needs silt to receive a descent deal of attention. Silt grains’ texture, size, sorting, and crystalline structure have prime control on their interaction with clay, chemicals, and capillary water bonds. Silts’ resistance against skeletal forces is also a factor of grains’ surficial and internal properties. (i.e the main supporting units for open spaces within the soil’s structure). As such, the purpose of this paper is to make a contribution to that understanding, and to look at the quartz size reduction in the sedimentary environment and the controls involved. Silt is formed through size reduction mechanisms. Jefferson et al. (1997) discussed a set of natural geochemical controls in silt formation from quartz-bearing igneous and metamorphic rocks.

These controls commenced with an initial transformation of ‘high quartz (β-quartz)’ into the more densely packed ‘low quartz (α-quartz or ordinary quartz)’ upon cooling to the hydrothermal temperatures in a granitic system. β-quartz forms after slow crystallization of siliceous (SiO₂-rich) magma. Sorby (1858) made a detailed study of quartz origin, and implicated the liquid inclusions in quartz to a history of slow crystallization of siliceous magma of granite at low heat (i.e. to a degree beyond 573 °C) but under great pressure. The quartz product is in fact a part of a coarse eutectic of quartz and feldspar (Smalley, 1966).

This eutectic reaction which delivers two new phases from one original phase leaves its footprints as that showed later herein (se Fig. 4 in section 4). Further temperature decrease allows the structure of high quartz to distort; such that 6-fold and 3-fold screw axes (60° and 120° inclination) change into 3-fold screw axes (60° inclination). Oxygen-Silicon bonds kinks and bends, which provides a more densely packed assemblage. The transformation from ‘high quartz’ to ‘low quartz’ is displacive (i.e. no bond breakage occurs), but the angle between oxygen bonds’ change. This causes contraction in the crystal. Contraction induces tensile stresses, normal to the c-axis (about which quartz contracts). These stresses fracture the crystal or induce a defect plane along the c-axis (Smalley, 1966). The defects led to crushing, delivering
600 µm modal particles into the sedimentary system. According to Blatt (1970), this 600 to 700 µm quartz was further crushed by 90%. The breakage resulted in a pronounced mode size of 60µm. The 60 µm silt was further crushed into a pronounced mode size of 20 to 60 µm (Kumar et al., 2006) and then to 20 to 30 µm (Jefferson et al., 1997). A control is shall exist on the breakdown process under moderate natural stresses (Jefferson et al., 1997).

Mechanism of quartz size reduction has been explained in a fractal framework (Hyslip and Vallejo, 1997; Mandelbrot, 1983). The use of fractal concept allows the simultaneous quantification of fragmentation and grain size distribution (Hyslip and Vallejo, 1997). Fragmentation (i.e. quartz size reduction) is a scale invariant natural process (Smalley et al., 2005), which is conventionally quantified by means of fractal concept (Turcotte, 1986). Fractal is basically a power law relation between number (particles’ population by mass) and size (particle’s diameter). Central to the fractal concept is the fractal dimension, which is a measure of the fracture resistance properties of dispersed systems (Brown et al., 1996), such as the crystalline defects in quartz sand and silt. A fractal dimension is a ratio providing a statistical index of complexity comparing how detail in a pattern (strictly speaking, a fractal pattern) changes with the scale at which it is measured. It has also been characterized as a measure of the space-filling capacity of a pattern that tells how a fractal scales differently than the space it is embedded in; a fractal dimension does not have to be an integer.

Lu et al. (2003) used the particle size distribution data to characterise the fractal properties of Leighton Buzzard sand. They assumed a uniform shape of particles, which is arguable in loess soils (Howarth, 2010; Rogers and Smalley, 1993). They then used the Schuhmann’s distribution (Fuerstenau, 2003) accompanied with (Turcotte, 1986) relation (between the fractal dimension and distribution index as discussed in section 4) and successfully described the fragmentation events. Fractal dimension however should be derived separately for clay and quartz minerals (Wang et al., 2008), due to the different origin of primary
and clay minerals (Posadas et al., 2001). This however does not apply to the clean crushed Leighton Buzzard sand, as this material contains no mineralogical gradient among its size scales.

The present empirical work examines the micromechanics of sand-to-silt size reduction in the quartz material. A series of grinding experiments was performed on Leighton Buzzard Lower Greensand using a high-energy disc mill. Analogous conditions to glacial abrasion are reportedly provided in disc mills due to the combined abrasion between grains’ asperity tips, and also between grains and rotating smooth tungsten carbide pestle. The grinding time and energy input were varied. Breakage pathway was inspected via Laser diffraction spectroscopy and transmission light microscopy. Arithmetic fractal measures to describe the breakage process were recorded. These included fractal dimension, relative breakage index, maximum grain size, pronounced mode size and sorting. Results from the grinding experiments together with the microscopy examinations were utilised to derive a timeline for the sand-to-silt size reduction phenomena.

2. Current Understanding of Silt Pedogenesis

Silt is a product of events in Peridesert, Perimountain, and Periglacial environments (Smith et al., 2002). Peridesert silt is generated from chemical and salt weathering (Pye, 1995), temperature fluctuations (Smalley et al., 2001) and seasonal wetting/drying and heating/cooling (Smith et al., 2002). Perimountain silt is generated from cold weathering (Zourmpakis et al., 2003) and frost shattering (Wright et al., 1998). Periglacial silt is produced from glacial grinding (Smalley et al., 2005) of granitic (Sorby, 1858) beds of glaciers. Less appreciated disintegrating processes include: sub-aerial and fluvial transport actions (Smalley et al., 2006 a), loessification (i.e. in-situ dry weathering on carbonate rich parent material that originally was deposited as alluvium on flood plains during Pleistocene – see Russell (1944) and Pecsi (1990)) and dry climate weathering (Assallay et al., 1996), desertification (Qiang et al., 2010), and volcanic
actions (Pouclet et al., 1999). However, glacial grinding (Periglacial) is widely accepted as the main source of present-day silt (Smalley et al., 2006a).

2.1 Geological Controls and Sand-to-Silt Size Reduction

The significance of internal weakness in quartz was first scientifically described by (Moss, 1966). In the line of an earlier research work of Wright and Larsen (1909), Moss (1966) classified the quartz into mature (intact) and defected types. Mature quartz has a background of less post-solidification modifications and fracturing-healing cycles, contains more non-undulatory extinction features and is less structurally damaged. This background grants mature quartz a considerable resistance to weathering, high durability and hardness. With non-intact quartz, cracks formed along the projected lines of internal defected planes, such as unopened healed fractures. In 1973, Moss showed the contribution of transient loads in grain breakage. He emphasised that the magnitude of applying static load might not be high enough to trigger the breakage. The transient load of the same magnitude, however, could crush the grain. He differentiated the grain breakage under transient loading environments by using the ‘fatigue fracturing’ term.’ Moss (1966) showed that controlled-rate cyclic loads of low order can crush the granitic quartz, while static loads of the same value may fail to break a similar grain. He then simulated a fluvial transport system by subjecting the granitic quartz to rotation in a steel drum containing water. Quartz was weakened in the long-term in transient loading environment (i.e. waves and streams). The ‘mean fragmentation load’ (i.e. load required to trigger the breakage) remarkably decreased by the end of the early rotation runs, highlighting the fatigue weakening of quartz grains. This agreed with Sharp and Gomez (1986) suggestion that grains break through both fatigue and surface fracturing. Fatigue effect was also addressed in Rabinowicz (1976), where certain textural features were linked with splitting events as stresses apply and release. The idea of silt production through fatigue fracturing in fluvial systems however was questioned in the work of Wright and Smith (1993). They reported small amounts produced in the range 2 to 20 µm silts by water-quartz abrasion. They showed the higher significance of impact-induced fracturing than
fatigue fracturing. They produced considerable mass of 20 to 60 μm grains by using rigid ceramic spheres in the rotating drum. In a different attempt, air-abrasion was simulated in Smith et al. (1991), by subjecting 350 to 500 μm sized Pannonian sand to air jet stream for 1 to 128 hours, generating remarkable contents of 20 μm grains in the first hour. Microscopic observations showed strong edge grinding (source of 20 μm fines) and appearance of fresh micro-fractures on large grains during the first hour. A secondary pronounced mode appeared after 16 hours at 20 to 40 μm, which then changed into 60 μm. Similar results were reported in Wright et al. (1998). The stepwise size reduction was in a good agreement with the fatigue fracturing concept. Jefferson et al. (1997) discussed the significance of quartz internal controls in air-abrasion processes. They quoted two similar wind tunnel experiments on two different sand materials (crystallographically perfect quartz in Kuenen (1960) and granitic quartz in Whalley et al. (1982)). Little silt was generated by crushing the crystallographically perfect quartz. Wright (1995) simulated the glacial grinding at the base of glaciers, by using a Bromhead ring shear. The ring shear was used for its closer approximation to subglacial environments than tumbling mills. After subjecting 250 to 500 μm freshly crushed (to simulate an identical stress history for entire grains according to Wright and Smith (1993)) Brazilian vein quartz to rotation under varying axial loads, Wright (1995) reported no evidence of fresh micro-cracks in grains. The little produced silt fitted well into the common loess size range, 20 to 60 μm. She then questioned the predominant contribution of quartz breakage at clast-bedrock interfaces in Pleistocene glaciers in silt production. Wright (1995) concluded a number of possible factors to explain the limited size reduction recorded in the majority of her experiments. She referred to findings of Bond (1952) and Rittinger (1867) that acknowledged a relatively higher energy required for fine sand-to-silt size reduction. This however was argued in (Jefferson et al., 1997; Kumar et al., 2006). These works showed significant contents of produced silt, after grinding 1 to 2 mm Leighton Buzzard quartz sand with a high-energy disc mill. They also showed two early and late periods of breakage at which easily breakable flawed and crystalline defected hard particles crushed, respectively. Assallay (1998) used a range of grinding
machines to simulate different breakage processes. He postulated that the end runner mill simulates the glacial grinding, ball mill simulates the particle impact, and compression machine simulates the natural compression forces. The ball mill (Assallay, 1998) is a much more gentle process than the disc mill. Assallay 1998 used the end-runner mill to grind the same material used in Jefferson et al. (1997). He reported that sand-sized fragments were crushed by 70% in size to 10 to 50 μm silts by the end of 2-hour grinding period. Kumar et al. (2006) examined the earlier work of Wright (1995) by repeating the same testing procedure on un-weathered vein quartz and marine Leighton Buzzard sand. They concluded that the little produced silt from un-weathered vein quartz is due to the absence of crystalline internal defects, and not a factor of the initial grain size.

2.2 Micromechanics of Sand-to-Silt Size Reduction

Moss (1973) developed one of the earliest quartz breakage models. He attributed the edge grinding to concentration of stresses at grains’ asperity tips, which provides greater chance for grain to split. Generated fine fragments fill the void spaces and overflow thereafter around survived larger particles. He postulated a higher chance of breakage in relatively large particles (in agreement with Sharp and Gomez 1986), due to presence of higher degrees of internal imperfections. Recently, Bolton (1999) investigated the micromechanics of crushable grains. He revealed that a grain’s resistant is a factor of the contact constraint conditions, particle size, and level of applied internal stress. He drew the attention to the higher tendency of smaller particles to split (in contrast with Moss (1973)). Small grains get trapped between neighbouring larger grains and attain the maximum chance of splitting in presence of two point contacts. Relatively finer grains carry the same force over a smaller surface area (Also see (Mitchell and Soga, 2005; Santamarina, 2003)) and therefore are subjected to higher levels of internal stresses. Coop and Altuhafi (2011) agreed with Bolton (1999), and emphasised that well sorted grains break more readily. This was ascribed to the increased number of grains’ contacts, which favours the edge grinding and fine crushing.
As a summary to the brief silt literature discussed above, it may be well to point out that there are questions of the quartz size reduction – a question of breakability of sand and silt and a question of controls on the size and population of the silt output. The present study allows for the sand-to-clay size reduction timeline to develop. Reading the microscopy examination results together with the measured fractal indexes on this breakdown timeline is expected to answer the two questions.

3. Testing Set-up and Material

Washed oven-dried Leighton Buzzard Lower Greensand quartz from Bedfordshire was mechanically ground in a high-energy Siebtechnik disc mill. The disc mill consisted of a barrel, which accommodates a ring and a tungsten carbide pestle. By means of predominantly horizontal vibrations, the material was ground by impact and friction. Milling took place from the impact between the pestle and the ring, and also between the ring and the inner wall of the barrel, crushing any material trapped in between. Furthermore, particle-to-particle abrasion and crushing of materials trapped underneath the pestle and the ring were other modes of milling. Analogous conditions to glacial abrasion are provided in disc mills (Jefferson et al., 1997) due to the combined abrasion between grains’ asperity tips, and also between grains and rotating smooth tungsten carbide pestle.

Sand was initially washed with tap water (and then Calgon) through a 63 μm sieve to remove the silt- and clay-sized fragments (fine quartz and clay minerals) before operating the grinding experiments. This allowed an accurate control on the mass of ‘silt’ production for a given energy input. A series of timed events with each grind time (up to 60 s) followed by a 30 s cooling period to prevent overheating. The discontinuous grinding regime (frequent stress application and release) controlled the internal fatigue stresses. In addition to the grinding duration, the magnitude of the energy input has a significant control on the silt output. A control on the feed mass (mass of the original sand inside the barrel) was used to imitate two input energies. The barrel could hold up to 360 g of Leighton Buzzard sand, the full capacity
of which were used initially. The sand mass was then reduced by 30%. This was felt to give greater impact energy as particles were allowed to move back and forth easier, in a given grinding timescale.

Using the disc mill allowed control on the length of grinding and cooling periods. The discontinuity of grinding allows the control on the fatigue stresses and therefore activation of crystalline discontinuities (Jefferson et al., 1997). However, this differs compared to the governing conditions under glaciers and other natural silt producing systems (Kumar et al., 2006).

After grinding, crushed material was carefully placed in sealed and labelled plastic bags to determine the particle size distribution (PSD) through the laser diffraction (LD) spectroscopy technique. Size analysis by LD however posed uncertainties (O’Hara-Dhand et al., 2013) with the population of <2.5µm grains (slight over-estimation) and the size of >50µm grains (under-estimation after the cross-checking data obtained from the standard gravity sedimentation – also see Pye and Blott (2004)). Uncertainties were deemed mainly due the coagulation of sub-rounded fine grains on the vibrating channel before the laser analysis and also the fact that LD machines assume all particles as perfect spheres. Other sources of errors could be the small sample sizes, which might not be a true representation of the test material (Cooper, 1998).

On the plus side, grading outputs produced by the LD technique are highly reproducible (Abbireddy and Clayton, 2009). Small samples taken after each set of grinding were viewed under optical microscopes (Leica DM LM optical and light transmission Zeiss Axioplan 2 petrological microscope). Light microscopy allows the study of crystalline features by transmitting the light through the grain samples. However, light microscopy imposes drawbacks to the results: the poor magnification and the low resolution. Furthermore, this approach does not allow the real-time observation of grains’ modification. However, the test material was ground in a closed system and no crushed material was removed from the system before a subsequent round of grinding. As the sand fraction was completely crushed after 180 s of grinding, the observed
surface imperfections in crushed grains were concluded to differ from textural features of grains at their initial stage.

4. Results and Discussion

4.1 Fractal Features of Breakdown Timeline

Hardin equation for relative breakage index (Hardin, 1985) was used to describe the size-reduction in quantitative terms. According to Hardin (1985) ‘Breakage potential’, $B_p$, is the area above the initial grading curve up to the ‘100% passing’ line, confined between the lower-bound 63 μm and the upper-bound maximum grains’ diameter (as for sand-size index) or the 2 μm lower- and 63 μm upper-bounds (as for silt-size index). The ‘Total breakage’, $B_t$, underlines the amount of crushing that the granular assembly has undergone and is represented by the area between the PSD curve pair of the initial and post-crushed state, while confined in the latter span of $B_p$. The relative breakage is defined as the ratio of $B_t$ over $B_p$. To derive the associated areas, the fitted functions of each grading curve were integrated along the particle size axis. Fig. 1 shows the relative breakage index for silt-sized and sand-sized scales against grinding time.

At sand-sized scale, there was a gradual rise in the breakage index in 0-120 s timescale from zero to just under 5%. Index then steeply increased by 95% to a peak of 100% in 120-180 s timescale. In other words, sand fragment was entirely crushed to silt-sized grains by 180 s of grinding. At silt-sized scale, breakage index slightly increased in 0-120 s timescale to just above 2%. It then sharply increased by 75% in 120-180 s timescale, before flattening out in 180-240 s timescale. The index improved thereafter, decelerated, levelling off, and finally recovered in the subsequent timescales. At 720 s of grinding, index hit the course
high of 86%. The early sharp increase in breakage index (in 120-180 s timescale), agrees with the activation of surface imperfections and existing micro-fractures (see Fig. 2-3).

In Fig. 4, Signs of crystalline gradients were spotted on the original sand grains. These may either implicate a history of fracturing-healing through the post-solidification period, or conditions under which quartz crystalized. Irregular V-shaped pits on grain's surface are potential lines of weakness through which splitting occurred after 180 s of grinding.

According to the PSD curve shown in Fig. 5, the early breakage improved the degree of uniformity by eliminating the sand-size fragments. Jefferson et al. (1997) ascribed the early gap between the PSD curves to the “simple breakage of the original flawed sand grains”. This agrees with observations made here in Fig 2.

Fig. 6 shows the progression of particle size distribution (PSD) curves with grinding time from 240 s to 720 s. The phi-scale divisions are added to the grading plots to address the silt's sub-divisions easier. For an increase in grinding time from 240 seconds to 300 seconds, a positively skewed gap appeared between the PSD curves at 4-5 Ø, which was then proceeded with a negatively skewed gap at 5-6 Ø. These gaps represented crushing events in very coarse to coarse silts. Almost the entire volume of very coarse silts was crushed at the 240-300 s grinding timescale.
Further increase in grinding time from 300 s to 360 s produced very little breakage and therefore led to a negligible gap between PSD curves at 5-6 Ø. Coarse silt grains were resistant against the impact energy, while crushing events in finer fragments continued at limited levels.

In the 600-720 seconds grinding timescale, the rate of breakdown surged at the 5-6 Ø (coarse silts). The positively skewed gap between PSD curves suggested higher degrees of particle breakage in finer fragments.

A possible reason for lack of significant abrasion at 300-360 s timescale may be the mature state of material after 300 s of crushing. However, sets of fresh micro-fractures formed within the crystal of mature grains. Formation of fresh imperfections may probably a physical signature of fatigue fracturing under relatively moderate impact energies. The randomly selected grain at 300-360 s timescale (Fig. 7) shows signatures of fresh (sharp) parallel ridges on surfaces.

Higher degree of breakage in finer fragments at 240-300 s timescale (6-7 Ø) and at 600-720 s timescale reveals the tendency of sand-sized particles for continuous breakage.

The results presented above showed that size reduction in quartz is fractal phenomena, consisting of periods of breakage followed by periods of fatigue fracturing. Experiments also showed that both survived and broken grains continue to break and re-arrange along the period of impact energy application. A given impact energy level might not be sufficient enough to trigger breakage, but the prolonged application of the energy can potentially induce fresh surface imperfections along the internal crystalline defect planes.

These highlights that neither sand nor silt is the end-product of abrasion at a given energy level. The experimental work also showed that high energy earth-surface processes such as the glacial abrasion generates significant amounts of silt-sized particles (Fig. 8 – early gap between the PSD curves at 5-7 Ø,
the pronounced mode size of loess, the main constituent of which is silt). However, further size reduction continues under less efficient input energies (i.e. the damping events, to be discussed in 4.3) but prolonged duration.

[Figure 8]

4.2 Fractal Dimension on Breakdown Timeline

In Fig. 9, grinding time is plotted against $K_{100}$ (i.e. maximum particle size). The plot shows fractal features, in which periods of stability are followed by periods of downturn trends. Declining trends however reduced in gradient with grinding time.

Largest particles survived within the first 120 s of grinding. Maximum diameter then plummeted at 180 s grinding time. Microscopic inspections suggested that this probably occurred due to the breakage of defected sand particles along lines of weakness (Fig. 2-4). The second period of resistance (little breakage) appeared at 180-240 s timescale before the second breakage event at 240-300 s timescale. This was then followed by the third period of resistance and the third breakage event.

[Figure 9]

At 120-180 s timescale, sand-sized scale breakage index hit 100% (Fig. 5), indicating the transition of entire sand-fraction into silt (Fig. 1) material. The significance of this transition appeared in the plummeted ‘maximum grain size’ by 96.8% (Fig. 9 – in agreement with Blatt (1970)). Breakdown in larger grains was more pronounced than in finer grains (Fig. 8). Microscopic examinations revealed chevron-shaped cracks on the surface of randomly selected fine sand, after 120 s of grinding (Fig. 10 – in agreement with Smith et al. (1991) and also with Krinsley and Doornkamp (1973) images 55 and 56). As sand-fraction faded by an increase in grinding time to 180seconds, splitting possibly occurred by exploitation of existing micro-fractions (consistent with Cheng (2004)). The next grinding timescale (180-240 s) could be regarded as a
period of grains’ resistance against the applying energy. This probably occurred due to the establishment of elevated number of contact points between grains (55 μm) and several edge-grinded finer particles (Fig. 8). The enhanced lateral confinement for grains allowed a better resistance against the applying energy. Further increase to the grinding time (to 300 s) however led to the breakdown of very coarse to coarse silt grains (4-6 Ø – see Fig. 6) and the second major drop in the maximum grain size (K100) as shown in Fig. 9. K100 flattened out thereafter at 300-360 s, and then marginally fell at 360-600 s timescale. The sequences of decreasing-plateau trends revealed the fractal characteristics of K100. Sequences also revealed the continuous breakage of large grain, although these grains survived splitting at certain previous grinding timescales due to the increasing number of lateral support from finer grains.

[Figure 9]

[Figure 10]

The step-wise particle breakage can be explained via the fractal dimension. This is drawn here from the power law exponent of Schuhmann distribution (Eq. 1).

\[ P = \left( \frac{S}{K_{100}} \right)^{n_s} \]

(Eq. 1)

Where ‘P’ is the passing percent (by mass) through sieve size ‘S’. The ‘index of uniformity’, \( n_s \), can be demonstrated by the slope of PSD fitting line on a double logarithm plot of cumulative passing percentage versus normalized nominal diameter (i.e. diameter divided by \( K_{100} \)). The fractal dimension is then derived from the index through Eq. 2 (Lu et al., 2003).

\[ D = 3 - n_s \]

(Eq. 2)
Fig. 11a shows the double logarithm plot of cumulative passing percentage versus normalized particles’ size. Graph shows an inverse relationship between Index of uniformity and grinding time. Fig. 10b shows the plot of fractal dimension against grinding time.

As shown in Fig. 11b, there was a steep rise in fractal dimension (i.e. poor sorting, well grading) in 0-180 s timescale from 1.7 to a peak of 2.6. Fractal dimension then remained constant throughout the 180-720 s timescale, with slight turbulence between 2.3 and 2.6. The plateau trend was probably due to the loss of energy efficiency through the grinding time. The declining trend of energy efficiency is due to the constant energy input through the experiment and the rising trend of fines population. As discussed earlier, fines provide lateral supports to relatively coarser grains and therefore damp the energy. Damping is also improves by the formation of platy crushed grains, and through enhanced degrees of lateral support to survived grains.

Fractal dimension was derived between 1.7 to 2.6, which generally conforms to the range of 2.2 to 2.6 reported earlier in Lu et al. (2003) for same material. Fractal dimension was then formulated as a function of grinding time. An inverse of hyperbola function was used so to match the earlier work of (Lu et al., 2003) on the same test material (Eq. 3). Levenberg-Marquardt algorithm in non-linear regression was applied to derive the ‘a’ and ‘b’ coefficients. Regression resulted the 0.384 value for ‘a’ and ‘7.312’ value for ‘b’ coefficients. The formulation then gave a theoretical maximum value of 2.51 for fractal dimension, as grinding time tending to infinity. ‘From the quartz size-reduction perspective, the above results suggested that that sand and silt with defects are inherently breakable materials for fractal dimensions less than an intrinsic maximum.

\[
D = \frac{t}{at + b}
\]

(Eq. 3)

4.3 Mode-size Distribution on Breakdown Timeline
The Leighton Buzzard sand possess an observed mode at around 100-400 µm (Fig. 12-a) and a secondary pronounced mode at around 40-60 µm. This corresponds well to the Quartz sand modal distribution in Assallay et al. (1998).

By the end of the initial grinding for 120 s, the original four mode sizes slightly shifted to left (Fig. 12-b). This gave rise to the population of grains at the silt-scale mode sizes (Fig. 12-b).

Further grinding to 180 s led to the disappearance of sand-scale mode size at the expense of an increased population of grains at silt-scale mode sizes (Fig. 12-c). An increase in the grinding time to 300 seconds faded the first pronounced silt-scale mode size (4-5 Ø or 31 to 62 µm – see Fig. 12-d). This can be viewed in conjunction with increasing breakage index at 240-300 s timescale (Fig. 1) and decreasing K100 at the same timescale (Fig. 9).

Results showed that all modes sizes at both sand- and silt-sized scales shifted or faded, except with the 10-20 µm mode size, which was preserved through the entire grinding time. The 10-20 µm appeared as an intrinsic characteristic of the quartz, which was reproducible at varied grinding durations.

[Figure 12]

4.4 Energy Input on Breakdown Timeline

The input energy was varied by putting a control on the mass of sand batch in mill’s barrel. For ten 60 s long discontinuous grinding intervals, the barrel’s feed mass was reduced by 30%. Increased input energy led to a poorly sorted silt output (Fig. 13a), maximal crushing of middle-sized grains and minimal crushing of relatively finer and coarser grains, and increase in population of 10-20 µm and 3vµm sized grains (Fig. 13b).

[Figure 13]
The maximum grain size ($K_{100}$) changed only marginally with increasing energy input. This probably was due to the insufficient increase in prompted energy to break the 32 µm silts, or due to the enhanced population of edge-grinded materials which in turn improved the confinement (or damping) around the 32 µm silt grains.

4.5 Discussion

The sand-to-silt approach is granitic quartz is examined. To enable an objective assessment of micromechanics of size reduction, measurements of particle and mode size distribution, fractal indexes and micromorphological signatures were made. The crushing approach was probed through varied grinding time at a constant energy input, as well as varied energy input at constant grinding time. The close inspection of sorting, mode sizes, grains’ population at mode sizes, fatigue stresses, and energy efficiency would suggest that:

1. The sand-to-silt size reduction pathway in Quartz possesses fractal properties (in sorting, maximum particle size, fractal dimension, and mode size distribution). Light transmission microscopy examination of sand- and silt-sized quartz samples before crushing revealed evidenced of surface imperfections and internal planes of varied taxonomy. As these were observed in grains after crushing, the surface and internal defects appears to be fractal features. This however slightly defers with the findings of Moss (1973) and Sharp and Gomez (1986). Therefore, the breakdown process explained here will also represent the size reduction mechanisms in coarser particles of debris at the base of glaciers.

2. Sand and silt are not final resistant products of the glacial comminution. Also, they are not the products of two mechanisms. These are in contrast with some earlier works (Rogers et al., 1963, Haldorsen, 1981, Wright, 1995), but in agreement with another line of works (Jefferson et al., 1997, Cheng, 2004, Kumar et al., 2006). This is because of the differences between crystallographically pure quartz and that with internal defects.
3. Results showed that there needs to be a prolonged application of moderate and declining energy (or alternatively application of a sudden but considerable energy) to crush a quartz fine sand or coarse silt to finer fragments. In other words, grain breakdown is not necessarily an energy-dependent process. Results also demonstrated that fatigue fracturing may occur either through prolonged stressing or transient stressing in a relatively shorter period. This agrees with the earlier works of Moss (1966) and Rabinowics (1976). This however argues the findings by Rittinger (1867) and Bond (1952) reported in Wright (1995).

4. Results showed an intrinsic pronounced mode size of 10-20 µm in crushed material at varied grinding timescales. It is most probable that the internal crystalline defects have some control on the size of the silt output.

5. The sand-to-silt approach affects the relatively larger grains more than finer grains. Further crushing of the silt fragment via increased energy input re-distributed grains population evenly without changing the maximum grains size. The approach affects the middle-sized grains more than the relatively finer-sized and coarse-sized fragments.

6. Conclusions

Arithmetic fractal measures to account for quartz breakage, including fractal dimension, relative breakage index, maximum grain size, pronounced mode size and sorting were made to examine the size reduction timeline. The many geomorphic, climatic, and geologic controls on the silt formation were briefly reviewed. A group of works which questioned the quartz continuous breakdown within the sedimentary cycle was here spent more consideration. In short, the present work showed seven key results below.

Quartz breakdown is a fractal phenomenon. Sand and silt are not the products of two mechanism, even though certain controls brought the sand into the sedimentary system while other controls operated on sand to bring the silt into the sedimentary system. This is generally in agreement with the early idea of
Wentworth (1933). Whether the universal continental silt is more a product of glacial abrasion or
sediment transport mechanism remains to be determined, but this research suggested that sand and silt
are not final resistant products of abrasion. The current research also showed that the crystallographically
defected sand and silt are inherently breakable materials for fractal dimensions less than an intrinsic
maximum. Grain breakdown is not necessarily an energy-dependent process. Internal defects in quartz
are independent from quartz size and the energy input. Experiments affirmed that a control exists which
delivers significant contents of particles in the 10-20 μm size. This control is independent from energy
input, energy duration, and grains’ starting size.

7. References

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Figure Captions

Fig 1 The Hardin’s relative breakage index (i.e. how far the grains are crushed in the scale of 0 to 1) against grinding time for silt-sized and sand-sized grains

Fig 2 Petrological microscopy image of a silt grain with surficial evidence of a fracture: a possible unopened healed micro-crack

Fig 3 Transmission light microscopy image of a sub-angular platy crushed quartz grain with surficial evidence of an internally crystalline defect plane: an inclined cleavage plane inside the grain

Fig 4 Crystalline gradients and surface imperfections under transmitted light for randomly selected silt before grinding

Fig 5 Particle size distribution curves of original sand material as well as ground materials (i.e. crushed for 120s to 720 s)

Fig 6 Particle size distribution curves of ground quartz material: crushed for 240 s, 300 s, 360 s, and 720 s

Fig 7 Sharp parallel ridges on a fine silt’s surface, indicating the possible fresh imperfections at the 300-360 s grinding timescale

Fig 8 Particle size distribution curves of ground quartz material: the significance of size reduction as grinding time increased from 120 s to 240 s

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The significance of the drop in maximum particle size ($K_{100}$) with an increase in the grinding time from 120 s to 240 s. The fractal pattern of decreasing $K_{100}$ with the impact energy input.

Chevron-shaped cracks on fine sand (120 s grinding).

The inverse relationship between the index of uniformity (i.e. slope of the PSD curve) and grinding time.

The sharp change in the fractal dimension through the early (i.e. under relatively low energy input) sand-to-silt transition and the significance of loss of impact energy efficiency due to the soared population of crushed fine particles.

Mode size distribution curve of the original and grinded materials (a) primary pronounced modes in the original sand (b) initial grinding and change in the population of the grains of pronounced mode sizes (c) the fully sand-to-silt transition after an increase in grinding time (d) the crushing of coarse silt into fine silt after a further increase in grinding time.

Loss in middle-sized silt grains and survival of fine and coarse sized silt grains upon an increase in the impact energy.